Characterization of two-dimensional Euclidean Landau states by coherent state transforms

2004 J. Phys. A: Math. Gen. 37 4813

(http://iopscience.iop.org/0305-4470/37/17/011)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **37** (2004) 4813–4819 PII: S0305-4470(04)69570-9

Characterization of two-dimensional Euclidean Landau states by coherent state transforms

Z Mouayn

Department of Mathematics, Faculty of Sciences and Technics (M'Ghila), Cadi Ayyad University, BP523, Béni Mellal, Morocco

E-mail: mouayn@math.net

Received 29 September 2003, in final form 26 February 2004
Published 14 April 2004
Online at stacks.iop.org/JPhysA/37/4813 (DOI: 10.1088/0305-4470/37/17/011)

Abstract

We construct a family of generalized coherent states attached to Landau levels of a charged particle moving in the two-dimensional Euclidean plane under a perpendicular uniform magnetic field. We prove that the ranges of the corresponding coherent state transforms coincide with spaces of bound states of the particle. This provides us with a new characterization of two-dimensional Euclidean Landau states.

PACS numbers: 02.20.−a, 03.65.Fd

1. Introduction

In recent decades considerable attention has been paid to the physics of two-dimensional (2D) quantum systems of charged particles, and, in particular, the Hall quantum effect [1]. The planar Landau problem [2] arises in the frame of quantum mechanics when a charged particle evolves under the influence of an external constant uniform magnetic field perpendicular to the Euclidean plane. This problem has been generalized to curved two-dimensional surfaces with a normal stationary magnetic field, such as the hyperbolic half-plane [3].

In a previous work [4], we were concerned with a family of generalized coherent states obtained by means of a square integrable representation of the group of affine transformations of the real line. This representation was realized on the Hilbert space of square integrable functions on the real positive half-line. We have proved that the ranges of the corresponding coherent state transforms coincide with spaces of bound states of the Landau Hamiltonian on the hyperbolic half-plane.

In this paper, we deal with an analogous question in the context of the two-dimensional Euclidean plane as configuration space and for the planar Landau Hamiltonian on it. Indeed, we construct for each Landau level a set of generalized coherent states obtained by displacing Gaussian–Hermite functions via operators of a unitary irreducible representation of the

Heisenberg–Weyl group. This allows us to characterize two-dimensional Euclidean Landau states as coherent state transforms of square integrable functions on the real line.

This paper is organized as follows. In section 2, we recall briefly some required facts on the planar Landau problem. Section 3 deals with the Perelomov presentation of constructing generalized coherent states via representations of the Heisenberg–Weyl group. In section 4, this formalism is applied so as to obtain a set of generalized coherent states attached to Landau levels. In section 5, we establish a characterization theorem for spaces of Euclidean Landau states by means of the corresponding coherent state transforms. Section 6 is devoted to some concluding remarks.

2. Spaces of 2D Euclidean Landau states

The Hamiltonian operator describing a particle of charge e and mass m_* which lives on the Euclidean xy-plane and interacting with a perpendicular constant homogeneous magnetic field is given by

$$H = \frac{1}{2m_*}\left(-\mathrm{i}\hbar\nabla - \frac{e}{c}\mathbf{A}\right)^2 \tag{2.1}$$

where $\hbar$ denotes Planck's constant, c is the light velocity and $\mathrm{i} = \sqrt{-1}$ stands for imaginary unit. We denote by $B > 0$ the strength of the magnetic field and we make use of the symmetric gauge $\mathbf{A} = -\frac{1}{2}\mathbf{r}\times\mathbf{B} = \left(-\frac{B}{2}y, \frac{B}{2}x\right)$, $\mathbf{r} = (x, y) \in \mathbb{R}^2$. For the sake of simplicity, we employ the units $m_* = e = c = \hbar = 1$ in (2.1). Therefore, we consider the Landau Hamiltonian

$$H_B := \frac{1}{2}\left[\left(\mathrm{i}\frac{\partial}{\partial x} - \frac{B}{2}y\right)^2 + \left(\mathrm{i}\frac{\partial}{\partial y} + \frac{B}{2}x\right)^2\right] \tag{2.2}$$

acting in the Hilbert space $L^2(\mathbb{R}^2, \mathrm{d}x\,\mathrm{d}y)$. It is well known that the spectrum $\sigma(H_B)$ of the operator H_B in (2.2) consists of eigenvalues of infinite multiplicity (Landau levels) of the form

$$E_n^B := \left(n + \tfrac{1}{2}\right)B \qquad n = 0, 1, 2, \ldots. \tag{2.3}$$

We let P_n denote the orthogonal projection operator onto the eigensubspace $\mathcal{E}_n^B(\mathbb{R}^2) := P_n(L^2(\mathbb{R}^2))$ of the operator H_B, which corresponds to the eigenvalue E_n^B. Note that spaces $\mathcal{E}_n^B(\mathbb{R}^2), n = 0, 1, 2, \ldots,$ consist of bound states of the particle. The operators $\{P_n\}$ are connected with the resolvent operator of H_B through the relation

$$(H_B - E)^{-1} = \sum_{n=0}^{+\infty}\frac{P_n}{(n+1/2)B - E} \qquad E \in \mathbb{C}\backslash\sigma(H_B). \tag{2.4}$$

The Green function (resolvent kernel) of H_B is given by ([5], p 215)

$$\begin{aligned} G_B(\mathbf{r}, \mathbf{r}'; E) = {}& \frac{1}{2\pi}\Gamma\left(\frac{1}{2} - \frac{E}{B}\right)\exp\left(-\frac{\mathrm{i}B}{2}\mathbf{r}\wedge\mathbf{r}' - \frac{B}{4}\|\mathbf{r} - \mathbf{r}'\|^2\right) \\ & \times\Psi\left(\frac{1}{2} - \frac{E}{B}, 1, \frac{B\|\mathbf{r} - \mathbf{r}'\|^2}{2}\right) \end{aligned} \tag{2.5}$$

where Γ is the Euler gamma function, $\mathbf{r}\wedge\mathbf{r}' := xy' - x'y$ and Ψ is the confluent hypergeometric function of the second kind. The latter decomposes into a series ([6], p 92) as

$$\Psi(a, c, \theta) = \frac{\theta^{1-c}}{\Gamma(a - c + 1)}\sum_{j=0}^{+\infty}\frac{1}{j + a - c + 1}L_j^{(1-c)}(\theta) \qquad c > \frac{1}{2} \qquad \theta > 0 \tag{2.6}$$

where $L_j^{(1-c)}(\cdot)$ is the generalized Laguerre polynomial [7]. Therefore, by (2.5) and (2.6) one obtains from (2.4) the matrix representation of the projection operators P_n as

$$P_n(\mathbf{r}, \mathbf{r}') = \frac{B}{2\pi} \exp\left(-\frac{\mathrm{i}B}{2}\mathbf{r} \wedge \mathbf{r}' - \frac{B}{4}\|\mathbf{r} - \mathbf{r}'\|^2\right) L_n^{(0)}\left(\frac{B}{2}\|\mathbf{r} - \mathbf{r}'\|^2\right). \tag{2.7}$$

The latter turns out to be the reproducing kernel of the Hilbert space $\mathcal{E}_n^B(\mathbb{R}^2)$ of Euclidean Landau states.

3. Generalized coherent states related to the Heisenberg–Weyl group

The concept of generalized coherent states related to the Heisenberg–Weyl group is used here following the Perelomov presentation (see [8], pp 226–7, and [9], section 1.1).

The simplest operators used in describing a quantum mechnical system with one degree of freedom are the coordinate operator q and the momentum operator p. Together with the identity operator I, they satisfy the commutation relations: $[p, q] = \mathrm{i}\hbar I$, $[p, I] = [q, I] = 0$, which characterize the well-known Heisenberg–Weyl Lie algebra. In this algebra, an element X of the form

$$X = u(\mathrm{i}p) + v(\mathrm{i}q) + t(\mathrm{i}I) \qquad u, v, t \in \mathbb{R} \qquad (\hbar = 1) \tag{3.1}$$

can be rewritten in terms of the annihilation and creation operators

$$a = \tfrac{1}{\sqrt{2}}(q + \mathrm{i}p) \qquad a^+ = \tfrac{1}{\sqrt{2}}(q - \mathrm{i}p)$$

as follows:

$$X = t(\mathrm{i}I) + \alpha a^+ - \alpha^* a$$

where

$$\alpha = \tfrac{1}{\sqrt{2}}(-u + \mathrm{i}v) \qquad \alpha^* = \tfrac{1}{\sqrt{2}}(-u - \mathrm{i}v). \tag{3.2}$$

By exponentiation, we obtain that

$$\exp(X) = \exp(\mathrm{i}tX)D(\alpha) \qquad D(\alpha) = \exp(\alpha a^+ - \alpha^* a).$$

The law of multiplication for the operators $D(\alpha)$ has the form

$$D(\alpha)D(\beta) = \exp(\mathrm{i}\,\mathrm{Im}\,\alpha\beta^*)D(\alpha + \beta). \tag{3.3}$$

As a consequence of (3.3), the operators $T(t, \alpha) := \exp(\mathrm{i}t)D(\alpha)$ form a unitary irreducible representation (UIR) of the Heisenberg–Weyl Lie group $\mathbb{W}_1$ whose underlying manifold is $\{g = (t, \alpha), t \in \mathbb{R}, \alpha \in \mathbb{C}\} = \mathbb{R} \times \mathbb{C}$.

Remark 3.1. If $|\phi_0\rangle$ is an arbitrary vector in the representation Hilbert space $L^2(\mathbb{R})$, one can see that the state corresponding to $|\phi_0\rangle$ is stable only under the operators of the form $T(t, 0)$. This is equivalent to saying that the isotropy subgroup of the state $|\phi_0\rangle$ is $\{(t, 0), t \in \mathbb{R}\}$.

Applying the UIR $T(g) = T(t, \alpha)$ to a vector $|\phi_0\rangle$ of $L^2(\mathbb{R})$, we obtain a set of states

$$|\alpha\rangle := D(\alpha)|\phi_0\rangle \qquad \alpha \in \mathbb{C}. \tag{3.4}$$

In view of remark 3.1, different α correspond to different states. The set $\{|\alpha\rangle\}$ is a system of generalized coherent states (GCS) of type $\{T(g), |\phi_0\rangle\}$, which satisfies, among other properties, the following ([9], p 15):

$$\langle\psi, \psi\rangle = \int_{\mathbb{C}} \mathrm{d}\mu(\alpha)\langle\alpha|\psi\rangle\langle\psi|\alpha\rangle \qquad \psi \in L^2(\mathbb{R}) \tag{3.5}$$

$\mathrm{d}\mu(\alpha)$ being the ordinary Lebesgue measure on $\mathbb{C} = \mathbb{R}^2$.

4. Generalized coherent states attached to Landau levels

For our purpose, we shall use the real variables (x, y):

$$x := -\frac{1}{\sqrt{B}}u \qquad y := -\frac{1}{\sqrt{B}}v.$$

(u, v) are the real variables introduced in (3.1) and connected to (α, α^*) by (3.2). In terms of (x, y) the action of the operator $D(\alpha) = D(x, y)$ on functions $\psi \in L^2(\mathbb{R}, d\xi)$ takes the following form (see [9], p 13):

$$D(x, y)[\psi](\xi) = \exp\left(\mathrm{i}\frac{B}{2}xy\right)\exp(-\mathrm{i}\sqrt{B}y\xi)\psi(\xi - \sqrt{B}x) \qquad \xi \in \mathbb{R}. \tag{4.1}$$

Now, to construct for a given Landau level E_n, a set of generalized coherent states, we choose as vector $|\phi_0\rangle$ the function Φ_n of $L^2(\mathbb{R}, d\xi)$ defined by

$$\Phi_n(\xi) := (\sqrt{\pi}2^n n!)^{-\frac{1}{2}} \exp\left(-\tfrac{1}{2}\xi^2\right) H_n(\xi) \qquad n = 0, 1, \ldots \tag{4.2}$$

where $H_n(\cdot)$ is the nth Hermite polynomial [7] and $\xi \in \mathbb{R}$. Note that the selected functions $\{\Phi_n\}$ in (4.2) form a total orthonormal system in $L^2(\mathbb{R}, d\xi)$ (cf [10], pp 77–9). Moreover, for $\lambda > 0$, the functions $\xi \to \sqrt{\frac{2}{\lambda}}\Phi_n\left(\sqrt{\frac{\lambda}{2}}\xi\right)$ are normalized eigenfunctions of the one-dimensional harmonic oscillator $\frac{d^2}{d\xi^2} - \frac{\lambda^2}{4}\xi^2$ with eigenvalues $\left(n + \frac{1}{2}\right)\lambda$, $n = 0, 1, 2, \ldots$.

According to (3.4), we can now define generalized coherent states (GCS) labelled by elements $(x, y) \in \mathbb{R}^2$ and $n = 0, 1, \ldots,$ as

$$|(x, y), B, n\rangle := D(x, y)[\Phi_n].$$

By (4.1) and (4.2), wavefunctions of these GCS have the form

$$\langle\xi|(x, y), B, n\rangle = (\sqrt{\pi}2^n n!)^{-\frac{1}{2}} \exp\left(-\mathrm{i}\sqrt{B}\xi y + \mathrm{i}\frac{B}{2}xy - \frac{1}{2}(\xi - \sqrt{B}x)^2\right) H_n(\xi - \sqrt{B}x).$$

For these GCS, equation (3.5) reads

$$\int_{\mathbb{R}^2} d\mu(x, y)\langle\psi|(x, y), B, n\rangle\langle\psi|(x, y), B, n\rangle^* = \langle\psi, \psi\rangle \qquad \psi \in L^2(\mathbb{R}, d\xi). \tag{4.3}$$

5. Coherent state transform and characterization theorem

Here, we start by noting that equation (4.3) says that the coherent state transform $\mathcal{W}_{B,n} : L^2(\mathbb{R}, d\xi) \to L^2(\mathbb{R}^2, d\mu)$ defined by

$$\mathcal{W}_{B,n}[\psi](x, y) := \int_{\mathbb{R}} d\xi\, \psi(\xi)\langle\xi|(x, y), B, n\rangle^*$$

is an isometrical embedding. Explicitly,

$$\mathcal{W}_{B,n}[\psi](x, y) = c_n \int_{\mathbb{R}} d\xi\, \psi(\xi) \exp\left(\mathrm{i}\sqrt{B}\xi y - \mathrm{i}\frac{B}{2}xy - \frac{1}{2}(\xi - \sqrt{B}x)^2\right) H_n(\xi - \sqrt{B}x)$$

where

$$c_n := (\sqrt{\pi}2^n n!)^{-\frac{1}{2}}.$$

Now, we shall make use of this constructed transform to establish a characterization theorem for spaces of Euclidean Landau states. Precisely, we obtain the following.

Theorem 5.1. *For $B > 0$ and $n = 0, 1, 2, \ldots,$ we have that*

$$\mathcal{W}_{B,n}[L^2(\mathbb{R})] = \mathcal{E}_n^B(\mathbb{R}^2).$$

To show that $\mathcal{W}_{B,n}[L^2(\mathbb{R})] \subset \mathcal{E}_n^B(\mathbb{R}^2)$ we need to compute the action of the Landau Hamiltonian H_B on $\mathcal{W}_{B,n}[\psi]$ for arbitrary function $\psi \in L^2(\mathbb{R})$. With the help of the identity ([7], p 1033)

$$\frac{\mathrm{d}}{\mathrm{d}u} H_n(u) = 2n H_{n-1}(u)$$

satisfied by the Hermite polynomial, a straightforward calculation gives

$$\begin{aligned}
&\frac{1}{2}\left[\left(\mathrm{i}\frac{\partial}{\partial x} - \frac{B}{2}y\right)^2 + \left(\mathrm{i}\frac{\partial}{\partial y} + \frac{B}{2}x\right)^2\right] W_{B,n}[\psi](x, y) \\
&\quad = \frac{1}{2}B^2x^2\mathcal{W}_{B,n}[\psi] - 2xB\sqrt{B}\mathcal{W}_{B,n}[\xi\psi] + B\mathcal{W}_{B,n}[\xi^2\psi] - \frac{\mathrm{i}B}{2}y\sqrt{B}\mathcal{W}_{B,n}[\xi\psi] \\
&\qquad + \frac{\mathrm{i}B^2}{2}xy\mathcal{W}_{B,n}[\psi] + \frac{\mathrm{i}nc_n}{c_{n-1}}B\sqrt{B}y\mathcal{W}_{B,n-1}[\psi] + \frac{\mathrm{i}B\sqrt{B}}{2}y\mathcal{W}_{B,n}[\xi\psi] \\
&\qquad - B\mathcal{W}_{B,n}[\xi^2\psi] + B\sqrt{B}x\mathcal{W}_{B,n}[\xi\psi] + \frac{2nc_n}{c_{n-1}}B\mathcal{W}_{B,n-1} + B\mathcal{W}_{B,n}[\psi] \\
&\qquad - \frac{\mathrm{i}B^2}{2}xy\mathcal{W}_{B,n}[\psi] + B\sqrt{B}x\mathcal{W}_{B,n}[\xi\psi] - B^2x^2\mathcal{W}_{B,n}[\psi] \\
&\qquad - \frac{2nc_n}{c_{n-1}}B\sqrt{B}x\mathcal{W}_{B,n-1}[\psi] - \frac{\mathrm{i}nc_n}{c_{n-1}}B\sqrt{B}y\mathcal{W}_{B,n-1}[\psi] + \frac{2nc_n}{c_{n-1}}B\mathcal{W}_{B,n-1}[\xi\psi] \\
&\qquad - \frac{2nc_n}{c_{n-1}}B\sqrt{B}x\mathcal{W}_{B,n-1}[\psi] - \frac{4n(n-1)c_n}{c_{n-2}}B\mathcal{W}_{B,n-2}[\psi] \\
&\quad = nBc_n\int_{\mathbb{R}} \mathrm{d}\xi\,\psi(\xi)\exp\left(\mathrm{i}\sqrt{B}\xi y - \mathrm{i}\frac{B}{2}xy - \frac{1}{2}(\xi - \sqrt{B}x)^2\right) \\
&\qquad \times 2(\xi - \sqrt{B}x)H_{n-1}(\xi - \sqrt{B}x) \\
&\qquad - 2n(n-1)Bc_n\int_{\mathbb{R}} \mathrm{d}\xi\,\psi(\xi)\exp\left(\mathrm{i}\sqrt{B}\xi y - \mathrm{i}\frac{B}{2}xy - \frac{1}{2}(\xi - \sqrt{B}x)^2\right) \\
&\qquad \times H_{n-2}(\xi - \sqrt{B}x) + \frac{1}{2}B\mathcal{W}_{B,n}[\psi].
\end{aligned}$$

By the recursion formula for the Hermite polynomial ([7], p 1033)

$$H_n(u) = 2uH_{n-1}(u) - 2(n-1)H_{n-2}(u)$$

we obtain that

$$H_B\mathcal{W}_{B,n}[\psi] = nB\mathcal{W}_{B,n}[\psi] + \tfrac{1}{2}B\mathcal{W}_{B,n}[\psi] = E_n^B\mathcal{W}_{B,n}[\psi].$$

Conversely, let $f \in \mathcal{E}_n^B(\mathbb{R}^2)$. We consider the function

$$\psi_f(\xi) := \frac{B}{2\pi}\int_{\mathbb{R}^2} \mathrm{d}\mu(x', y')f(x', y')\langle\xi|(x', y'), B, n\rangle \qquad \xi \in \mathbb{R}.$$

We shall calculate

$$\begin{aligned}
\mathcal{W}_{B,n}[\psi_f](x, y) &= \int_{\mathbb{R}} \mathrm{d}\xi\,\psi_f(\xi)\langle\xi|(x, y), B, n\rangle^* \\
&= \frac{B}{2\pi}\int_{\mathbb{R}} \mathrm{d}\xi\int_{\mathbb{R}^2} \mathrm{d}\mu(x', y')f(x', y')\langle\xi|(x', y'), B, n\rangle\langle\xi|(x, y), B, n\rangle^*.
\end{aligned}$$

Reversing the order of integration, we can write

$$\mathcal{W}_{B,n}[\psi_f](x,y) = \frac{B}{2\pi}c_n^2\int_{\mathbb{R}^2} \mathrm{d}\mu(x',y')f(x',y') \times \int_{\mathbb{R}} \mathrm{d}\xi \exp\left(-\mathrm{i}\sqrt{B}\xi y' + \mathrm{i}\frac{B}{2}x'y' - \frac{1}{2}(\xi-\sqrt{B}x')^2\right) H_n(\xi-\sqrt{B}x') \times \exp\left(\mathrm{i}\sqrt{B}\xi y - \mathrm{i}\frac{B}{2}xy - \frac{1}{2}(\xi-\sqrt{B}x)^2\right) H_n(\xi-\sqrt{B}x).$$

By using sucessive changes of variable ξ, we obtain

$$\begin{aligned}\mathcal{W}_{B,n}[\psi_f](x,y) &= \frac{B}{2\pi}c_n^2\int_{\mathbb{R}^2} \mathrm{d}\mu(x',y')f(x',y')\exp\left(-\frac{\mathrm{i}B}{2}(xy'-yx') - \frac{B}{4}(x-x')^2\right)\\ &\quad\times \int_{\mathbb{R}} \mathrm{d}\xi \exp(-\xi^2 + \mathrm{i}\sqrt{B}\xi(y-y'))H_n\left(\xi + \frac{1}{2}\sqrt{B}(x-x')\right)\\ &\quad\times H_n\left(\xi - \frac{1}{2}\sqrt{B}(x-x')\right)\\ &= \frac{B}{2\pi}c_n^2\int_{\mathbb{R}^2} \mathrm{d}\mu(x',y')f(x',y')\\ &\quad\times \exp\left(-\frac{\mathrm{i}B}{2}(xy'-yx') - \frac{B}{4}((x-x')^2+(y-y')^2)\right)\\ &\quad\times \int_{\mathbb{R}} \mathrm{d}\xi\, \mathrm{e}^{-\xi^2} H_n\left(\xi + \frac{1}{2}\sqrt{B}((x-x')+\mathrm{i}(y-y'))\right)\\ &\quad\times H_n\left(\xi - \frac{1}{2}\sqrt{B}((x-x')-\mathrm{i}(y-y'))\right).\end{aligned}$$

Making use of the identity ([7], p 838)

$$\int_{-\infty}^{+\infty} \mathrm{e}^{-u^2} H_j(u+\alpha)H_k(u+\beta)\,\mathrm{d}u = 2^k\sqrt{\pi}\,j!\beta^{k-j}L_j^{(k-j)}(-2\alpha\beta) \qquad j \leqslant k \tag{5.1}$$

for $j = k = n$, we obtain

$$\mathcal{W}_{B,n}[\psi_f](\mathbf{r}) = \int_{\mathbb{R}^2} \mathrm{d}\mu(\mathbf{r}')f(\mathbf{r}')\exp\left(-\frac{\mathrm{i}B}{2}\mathbf{r}\wedge\mathbf{r}' - \frac{B}{4}\|\mathbf{r}-\mathbf{r}'\|^2\right)\frac{B}{2\pi}L_n^{(0)}\left(\frac{B}{2}\|\mathbf{r}-\mathbf{r}'\|^2\right).$$

But the function of variables $(\mathbf{r},\mathbf{r}')$ in the integral above coincides with the reproducing kernel $P_n(\mathbf{r},\mathbf{r}')$ of the eigenspace $\mathcal{E}_n^B(\mathbb{R}^2)$, given in (2.7). Therefore, the last equation gives that $\mathcal{W}_{B,n}[\psi_f] = f$. It satisfies $\langle\psi_f,\psi_f\rangle_{L^2(\mathbb{R})} < +\infty$ since $\mathcal{W}_{B,n}$ is an isometry. We have then proved the inclusion $\mathcal{E}_n^B(\mathbb{R}^2) \subset \mathcal{W}_{B,n}[L^2(\mathbb{R})]$.

Before ending this section we should note the following:

(i) The form of the inverse coherent state transform from $\mathcal{E}_n^B(\mathbb{R}^2)$ into $L^2(\mathbb{R})$ is given by

$$f \to \frac{B}{2\pi}\int_{\mathbb{R}^2} \mathrm{d}\mu(x',y')f(x',y')\langle\xi|(x',y'),B,n\rangle.$$

(ii) For $m = 0,1,\ldots$ and $(a,b) \in \mathbb{R}^2$, the image of the GCS $|(a,b),B,m\rangle$ under the transform $\mathcal{W}_{B,n}$ can be calculated by making use of identity (5.1). Indeed, we obtain that

$$\begin{aligned}\mathcal{W}_{B,n}[|(a,b),B,m\rangle](x,y) &= \int_{\mathbb{R}} \mathrm{d}\xi\,\langle\xi|(a,b),B,n\rangle\langle\xi|(x,y),B,n\rangle^*\\ &= c_nc_m2^{\max(n,m)}\sqrt{\pi}\min(n,m)\end{aligned}$$

$$\times \exp\left(\frac{\mathrm{i}B}{2}(ay - xb) - \frac{B}{4}((x - x')^2 + (y - y')^2)\right)$$
$$\times \left(\frac{1}{2}\sqrt{B}((a - x)^2 + \mathrm{i}(b - y)^2)\right)^{\max(n,m)-\min(n,m)}$$
$$\times L_{\min(n,m)}^{(\max(n,m)-\min(n,m))}\left(\frac{B}{2}((a - x)^2 + (b - y)^2)\right).$$

(iii) The action of the Heisenberg–Weyl group on the Landau states space is given by means of the operators $\tau((x, y), t) : \mathcal{E}_n^B(\mathbb{R}^2) \to \mathcal{E}_n^B(\mathbb{R}^2)$, $((x, y), t) \in \mathbb{W}_1$, defined by $\tau((x, y), t) := \mathcal{W}_{B,n} o\, \mathrm{e}^{\mathrm{i}Bt} D(x, y) o \mathcal{W}_{B,n}^{-1}$, $D(x, y)$ are the operators given in (4.1). A direct computation gives

$$\tau((x, y), t)[f](x', y') = \exp \mathrm{i}B\left(t + \tfrac{1}{2}(xy' - x'y)\right) f(x' - x, y' - y)$$

for $f \in \mathcal{E}_n^B(\mathbb{R}^2)$ and $(x', y') \in \mathbb{R}^2$.

6. Concluding remarks

In our consideration of a charged particle moving in the two-dimensional Euclidean plane under the influence of a perpendicular uniform magnetic field, we have constructed for each Landau level a set of generalized coherent states by displacing Gaussian–Hermite functions via operators of a unitary irreducible representation of the Heisenberg–Weyl group. We have established that under the corresponding coherent state transforms, the images of the representation Hilbert space coincide with spaces of bound states of the particle. By this characterization, the coherent state method has provided us with a new way of looking at eigenspaces of the Landau Hamiltonian. In particular, the constructed coherent state transforms can be used to build two-dimensional Landau states from bound states of the one-dimensional harmonic oscillator.

Acknowledgments

I would like to thank the referees for their highly instructive remarks and suggestions. I am very grateful to my colleagues Dr M Daoud and Dr S Hilout for sending me valuable references. Many thanks also go to my friends Dr J Koubali and Dr I Tacat for their constant support and encouragement.

References

[1] Klitzing K V, Dorda G and Popper M 1980 *Phys. Rev. Lett.* **45** 494
[2] Landau L and Lifchitz E 1977 *Quantum Mechanics: Non-Relativistic Theory* (New York: Pergamon)
[3] Comtet A 1987 On Landau levels on hyperbolic plane *Ann. Phys.* **173** 185
[4] Mouayn Z 2003 *J. Phys. A: Math. Gen.* **36** 8071
[5] Chuburin Y P 2003 *Theor. Math. Phys.* **134** 212
[6] Tricomi F G 1954 *Funzioni ipergeometriche confliuenti* (Rome: Editioni cremonese)
[7] Gradshteyn I S and Ryzhik I M 1980 *Table of Integrals, Series and Products* (New York: Academic)
[8] Perelomov A 1972 *Commun. Math. Phys.* **26** 222
[9] Perelomov A 1986 *Generalized Coherent States and Their Applications* (New York: Springer)
[10] Titchmarsh E C 1950 *Introduction to the Theory of Fourier Integrals* 2nd edn (Oxford: Clarendon)